Test Prep 3 Worksheet 2

1. $a_0 = -2, a_{n+1} = 6a_n - \frac{8}{3}$

$a_1 = 6(-2) - \frac{8}{3} = -\frac{44}{3}$, $a_2 = 6(-\frac{44}{3}) - \frac{8}{3} = -\frac{272}{3}$

$a_3 = 6(-\frac{272}{3}) - \frac{8}{3} = -\frac{1640}{3}$, $a_4 = 6(-\frac{1640}{3}) - \frac{8}{3} = -\frac{9848}{3}$

2. $\lim_{n\to\infty} \frac{3n^2+ln(n)}{2n^2-ln(n)} = \frac{\infty}{\infty}$

$\stackrel{LH}{=} \lim_{n\to\infty} \frac{6n+\frac{1}{n}}{4n-\frac{1}{n}} = \frac{\infty}{\infty}$

$\stackrel{LH}{=} \lim_{n\to\infty} \frac{6-\frac{1}{n^2}}{4+\frac{1}{n^2}} = \frac{6}{4} = \frac{3}{2}$

3. $\lim_{n\to\infty} \frac{3n^3}{4n^4-4n^2+5} = \frac{\infty}{\infty}$

$\stackrel{LH}{=} \lim_{n\to\infty} \frac{9n^2}{16n^3-8n} = \frac{\infty}{\infty}$

$\stackrel{LH}{=} \lim_{n\to\infty} \frac{18n}{48n^2-8} = \frac{\infty}{\infty}$

$\stackrel{LH}{=} \lim_{n\to\infty} \frac{18}{96n} = \frac{18}{\infty} \to 0$

4. $R_n < \int_N^\infty \frac{1}{x^2+1}$

$R_n < \lim_{b\to\infty} \int_N^b \frac{1}{x^2+1}\,dx \to \lim_{b\to\infty} \tan^{-1}(x)\Big|_N^b \to \lim_{b\to\infty} \left[\tan^{-1}(b)-\tan^{-1}(N)\right]$

$\frac{\pi}{2} - \tan^{-1}(N) < 0.1 \to \frac{\pi}{2} - 0.1 < \tan^{-1}(N) \to \tan\left(\frac{\pi}{2}-0.1\right) < N \approx 9.967$

$N \geq 10$

5. $\sum_{n=1}^{\infty} \frac{1}{2^n-1}$

$b_n = \frac{1}{2^n-1} \geq \frac{1}{2^n} = a_n > 0$

$\geq \sum_{n=1}^{\infty} \frac{1}{2^n} = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$

$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r} \quad -1<r<1$

$\sum_{n=1}^{\infty} \frac{1}{2}\left(\frac{1}{2}\right)^{n-1} = \frac{1/2}{1-1/2}, \quad -1<\frac{1}{2}<1 \qquad \frac{1/2}{1/2} = 1 < \infty$

So, $0 < \sum_{n=1}^{\infty} \frac{1}{2^n} = 1 \leq \sum_{n=1}^{\infty} \frac{1}{2^n-1}$

So, the comparison test fails, we gain no information, and we try a different test (if told to do so)

6. $\lim_{n\to\infty} \frac{\left(\frac{3\sqrt{n}}{n^8+5}\right)}{\left(\frac{1}{n^{15/2}}\right)}$

$$\lim_{n\to\infty} \frac{n^{15/2}}{1} \cdot \frac{3n^{1/2}}{n^8+5} = \lim_{n\to\infty} \frac{3n^8}{n^8+5} = \lim_{n\to\infty} \frac{\cancel{n^8}(3)}{\cancel{n^8}\left(1+\frac{5}{n^8}\right)}$$

$$\lim_{n\to\infty} \frac{3}{1+\frac{5}{n^8}} = \frac{3}{1+0} = 3 \neq 0$$

7. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}3n}{5n+2}$ $\qquad b_n = \frac{3n}{5n+2}$

$$\lim_{n\to\infty} \frac{3n}{5n+2} = \lim_{n\to\infty} \frac{\cancel{n}\,3}{\cancel{n}\left(5+\frac{2}{n}\right)} = \frac{3}{5+0} = \frac{3}{5} \neq 0$$

So, the alternating series test fails, no information gained,

$\lim_{n\to\infty} \frac{(-1)^{n+1}3n}{5n+2}$ does not exist

So, since $\lim_{n\to\infty} \frac{(-1)^{n+1}3n}{5n+2} \neq 0$, the divergence test indicates $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}3n}{5n+2}$ diverges

8. $\sum_{n=1}^{\infty} \frac{\sin n}{n^3}$

$$\sum_{n=1}^{\infty} \left| \frac{\sin(n)}{n^3} \right| = \sum_{n=1}^{\infty} \frac{|\sin(n)|}{n^3}$$

$-1 \le \sin(n) \le 1$

$0 \le |\sin(n)| \le 1$

$$\le \sum_{n=1}^{\infty} \frac{1}{n^3} < \infty \quad p = 3 > 1$$

So, by the comparison test, since $\sum_{n=1}^{\infty} \frac{1}{n^3} < \infty$, then $\sum_{n=1}^{\infty} \left| \frac{\sin(n)}{n^3} \right| < \infty$

hence, $\sum_{n=1}^{\infty} \frac{\sin(n)}{n^3}$ is absolutely convergent

9. $\sum_{n=1}^{\infty} \frac{9^n}{(-2)^n n}$

$$\lim_{n\to\infty} \left| \frac{\frac{9^{n+1}}{(-2)^{n+1}(n+1)}}{\frac{9^n}{(-2)^n n}} \right| = \lim_{n\to\infty} \left| \frac{\cancel{9^{n+1}}\, 9^1}{\cancel{(-2)^{n+1}}(n+1)} \cdot \frac{\cancel{(-2)^n}\, n}{\cancel{9^n}} \right|$$

$(-2)^1$

$$\lim_{n\to\infty} \left| \frac{9n}{(-2)(n+1)} \right| = \lim_{n\to\infty} \frac{9n}{2(n+1)} = \frac{9}{2} \lim_{n\to\infty} \frac{n}{n+1} = \frac{9}{2} \cdot 1 = \frac{9}{2} =: L > 1$$

So, by the ratio test, $\sum_{n=1}^{\infty} \frac{9^n}{(-2)^n n}$ diverges