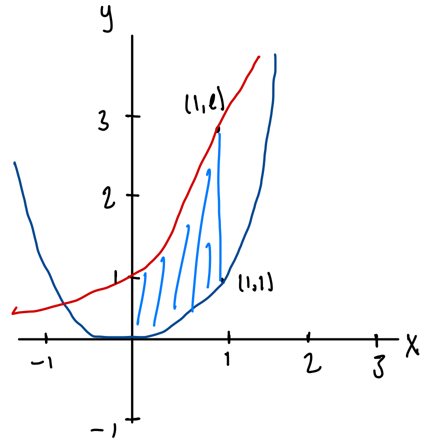
Areas Between Curves & Volumes

1. Set up an integral to find the area between the curves of $y = e^x$ and $y = x^6$

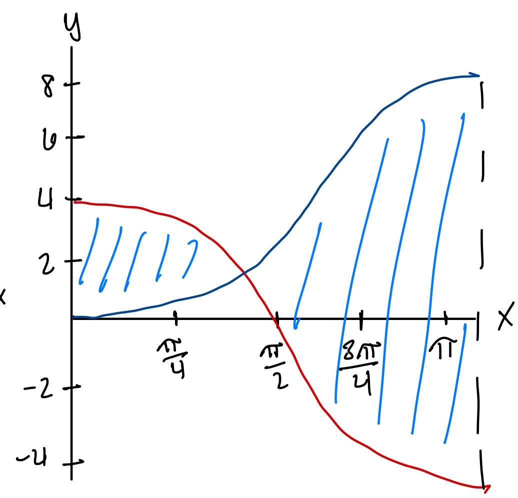
$$A = \int_{x=0}^{x=1} (y_T - y_B)\,dx$$



$$A = \int_0^1 (e^x - x^6)\,dx$$

$$= \left[e^x - \frac{1}{7}x^7\right]_0^1 = \left(e - \frac{1}{7}\right) - (1 - 0) = e - \frac{8}{7}$$

2. Sketch the graph and find the area between $y = 4\cos x$ and $y = 4 - 4\cos x$ from $0 \le x \le \pi$



$$4\cos(x) = 4 - 4\cos(x) \quad [0, \pi)$$

$$8\cos(x) = 4 \rightarrow \cos(x) = \frac{1}{2} \rightarrow x = \frac{\pi}{3}$$

$$A = \int_0^{\pi/3} [4\cos(x) - (4 - 4\cos(x))]\,dx + \int_{\pi/3}^{\pi} [(4 - 4\cos(x)) - 4\cos(x)]\,dx$$

$$= \int_0^{\pi/3} [8\cos(x) - 4]\,dx + \int_{\pi/3}^{\pi} [4 - 8\cos(x)]\,dx$$

$$= \left[8\sin(x) - 4x\right]_0^{\pi/3} + \left[4x - 8\sin(x)\right]_{\pi/3}^{\pi}$$

$$= \left(4\sqrt{3} - \frac{4}{3}\pi\right) - 0 + (4\pi - 0) - \left(\frac{4}{3}\pi - 4\sqrt{3}\right)$$

$$= 8\sqrt{3} + \frac{4}{3}\pi$$

3. Find the area of the triangle with vertices at (0,0), (5,3), (3,4)

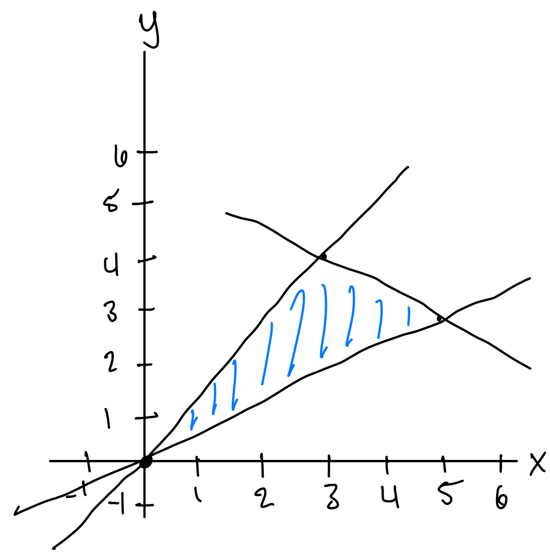
$\frac{3-0}{5-0} = \frac{3}{5} \rightarrow y = \frac{3}{5}x$ $\qquad$ $\frac{4-0}{3-0} = \frac{4}{3} \rightarrow y = \frac{4}{3}x$ $\qquad$ $\frac{4-3}{3-5} = -\frac{1}{2}x + \frac{11}{2}$

$$A = \int_0^3 \left(\frac{4}{3}x - \frac{3}{5}x\right)dx + \int_3^5 \left[\left(-\frac{1}{2}x + \frac{11}{2}\right) - \frac{3}{5}x\right]dx$$

$$= \int_0^3 \left(\frac{11}{15}x\right)dx + \int_3^5 \left(-\frac{11}{10}x + \frac{11}{2}\right)dx$$

$$\left[\frac{11}{30}x^2\right]_0^3 + \left[-\frac{11}{20}x^2 + \frac{11}{2}x\right]_3^5$$

$$\frac{33}{10} + \left(-\frac{55}{4} + \frac{55}{2}\right) - \left(-\frac{99}{20} + \frac{33}{2}\right) = \frac{11}{2}$$



4. Set up the integral for the volume of the solid given $x = \sqrt{6-y}$, y=0, x=0; about the y-axis

$$x = \sqrt{6-y} = 0 \rightarrow y = 6$$

$$V = \int_0^6 \pi (x)^2 \, dy$$

$$V = \int_0^6 \pi \left(\sqrt{6-y}\right)^2 dy$$

5. Find the volume of the solid, sketch the region and the disk or washer of $x = 2\sqrt{5y}$, x=0, y=3; about the y-axis

$X = 2\sqrt{5y} = 0 \rightarrow y = 0 \qquad y = \frac{x^2}{20}$

$V = \int_0^3 \pi \left(2\sqrt{5y}\right)^2 dy$

$= 4\pi \int_0^3 5y \, dy$

$= 4\pi \left[\frac{5}{2}y^2\right]_0^3 = 4\pi\left(\frac{45}{2}\right) = 90\pi$