Work & Average Value of a Function

1. How much work is done (in J) when a weightlifter lifts 210 kg from 1.5 m to 2.0 m above the ground?

$W = Fd$

$F = mg$

$F = 210\,kg\left(9.8\,\frac{m}{s^2}\right) = 2058\,N$

$W = 2058\,N\,(2m - 1.5m) = 1029\,N \cdot m = 1029\,J$

2. A variable force of $6x^{-2}$ pounds moves an object along a straight line when it is x feet from the origin. Find the work done (in ft-lb) in moving the object from x=1 ft to x=13 ft.

$W = \int_a^b f(x)\,dx$

$a = 1$ ft

$b = 13$ ft

$W = \int_1^{13} 6x^{-2}\,dx = 6\left[-x^{-1}\right]_1^{13}$

$= 6\left(-\frac{1}{13} + 1\right) \approx 5.54$ ft·lb

3. A force of 18 lb is required to hold a spring stretched 8 in beyond its natural length. How much work W is done in stretching it from its natural length to 10 in beyond?

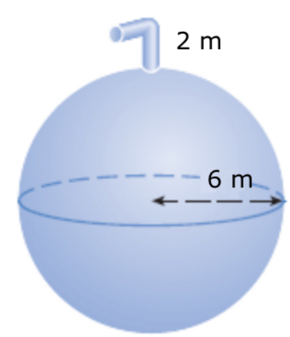
$f(x) = kx$ $\quad x = 8\text{ in} = \frac{2}{3}\text{ ft}$

$18 = \frac{2}{3}k \rightarrow k = 27\,\frac{lb}{ft} \rightarrow f(x) = 27x$

$W = \int_a^b f(x)\,dx$ $\quad a = 0\text{ in} = 0\text{ ft}$

$b = 10\text{ in} = \frac{5}{6}\text{ ft}$ $\quad W = \int_0^{5/6} 27x\,dx = \left[\frac{27}{2}x^2\right]_0^{5/6} = \frac{75}{8}\text{ ft}\cdot\text{lb}$

4. A tank is full of water. Find the work (in J) required to pump the water out of the spout. Use 9.8 m/s^2 for g and 1000 kg/m^3 for density of water.



$r = \sqrt{6^2 - y^2} = \sqrt{36 - y^2} \rightarrow r^2 = 36 - y^2$

$V = \pi r^2 \Delta y = \pi(36 - y^2)\Delta y$

$w = 9.8\frac{m}{s^2}\left(1000\frac{kg}{m^3}\right) = 9.8\times10^3\text{ N}$

$\Delta y = y + 8$

$W = \int_{-6}^{6} (9.8\times10^3)(y+8)\pi(36-y^2)\,dy = (9.8\times10^3)\pi\int_{-6}^{6} -y^3 - 8y^2 + 36y + 288\,dy$

$= (9.8\times10^3)\pi(2)\int_0^6 -y^3 - 8y^2 + 36y + 288\,dy = (9.8\times10^3)\pi(2)(8)\int_0^6 36 - y^2\,dy$

$= (156.8\times10^3)\pi\left[36y - \frac{1}{3}y^3\right]_0^6 = (156.8\times10^3)\pi(144) \approx 70934649\text{ J}$

5. Find the average value of $f(x) = 3x^2 + 4x$, $[-1,3]$.

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$a=-1$

$b=3$

$$f_{ave} = \frac{1}{3-(-1)}\int_{-1}^{3} 3x^2+4x\,dx = \frac{1}{4}\left[x^3+2x^2\right]_{-1}^{3}$$

$$= \frac{1}{4}(44) = 11$$

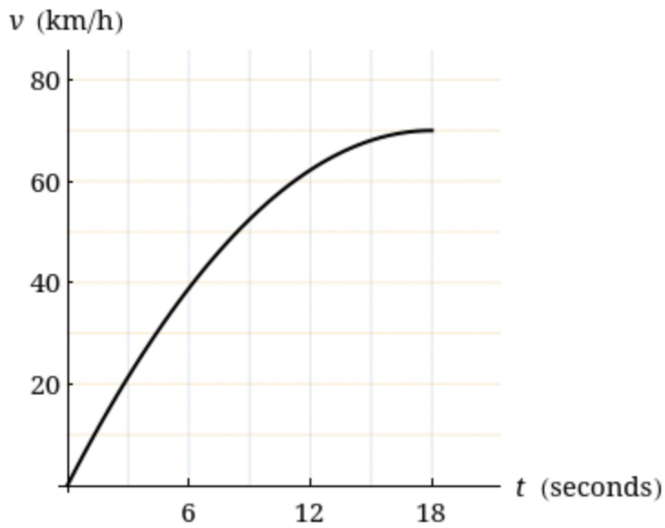
6. Find all numbers b so the average value of $f(x) = 2 + 10x - 9x^2$ on the interval $[0, b]$ is equal to 3.

$$\frac{1}{b}\int_0^b 2+10x-9x^2\,dx = \frac{1}{b}\left[2x+5x^2-3x^3\right]_0^b = 2+5b-3b^2$$

$$2+5b-3b^2=3 \rightarrow 3b^2-5b+1=0 \rightarrow b=\frac{5\pm\sqrt{(-5)^2-4(3)(1)}}{2(3)} = \frac{5\pm\sqrt{13}}{6}$$

both work because both are positive and must have $b>0$ for $[0,b]$

7. The graph of the velocity of a car that is accelerating is shown below.
   a. Use the midpoint rule with n=3 to estimate the velocity during the first 18 seconds.
   b. At what time was the instantaneous velocity equal to the average velocity?



a) $V_{ave} = \frac{1}{18-0}\int_0^{18} v(t)\,dt = \frac{1}{18} I$

$\Delta t = \frac{18-0}{3} = 6$

$I \approx M_3 = 6[V(3)+V(9)+V(15)] = 6[21+53+68] = 6(142) = 852$

$V_{ave} = \frac{1}{18}(852) \approx 47 \frac{km}{h}$

b) $v(t) = 47$

$t \approx 7.7s$

estimate by looking at the graph