Volumes, Cylindrical Shells

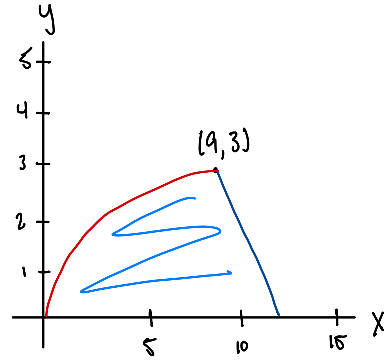
1. Set up the integral of the solid obtained by rotating about the x-axis the area under $y = \sqrt{x}$ and $y = 12 - x$ then find the volume of the solid

$y = \sqrt{x} \rightarrow x = y^2$

$y = 12 - x \rightarrow x = 12 - y$

$V = \int_0^3 2\pi y [x_R - x_L] dy$

$V = \int_0^3 2\pi y [(12-y) - y^2] dy$



$V = \int_0^3 2\pi (12y - y^2 - y^3) dy$

$= 2\pi \left[6y^2 - \frac{1}{3}y^3 - \frac{1}{4}y^4\right]_0^3$

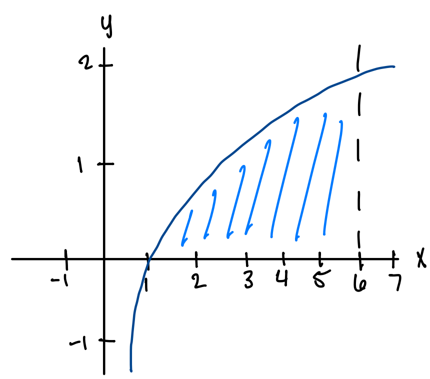
$= 2\pi \left[54 - 9 - \frac{81}{4} - 0\right] = 2\pi\left(\frac{99}{4}\right) = \frac{99}{2}\pi$

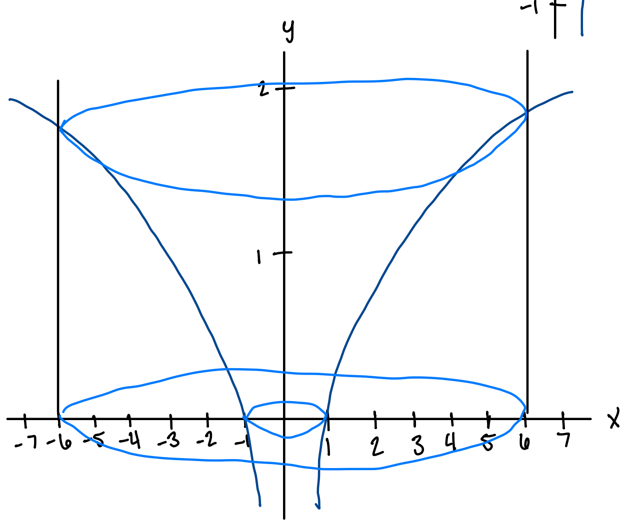
2. Set up the integral for the volume of the solid given $y = \ln x$, y=0, x=6; about the y-axis

$y = \ln(x) = 0 \rightarrow x = 1$

$V = \int_{x_1}^{x_2} 2\pi x (y) dx$

$V = \int_1^6 2\pi x (\ln(x)) dx$





3. Use cylindrical shells to find the volume of the solid rotated about the x-axis of $x = -3y^2 + 15y - 18$, x=0

$0 = -3y^2 + 15y - 18 \quad y = 2, 3$

$V = \int_2^3 2\pi y(-3y^2 + 15y - 18)\,dy$

$= 2\pi \int_2^3 (-3y^3 + 15y^2 - 18y)\,dy$

$= -6\pi \int_2^3 (y^3 - 5y^2 + 6y)\,dy$

$= -6\pi \left[\frac{1}{4}y^4 - \frac{5}{3}y^3 + 3y^2\right]_2^3$

$= -6\pi \left[\left(\frac{81}{4} - 45 + 27\right) - \left(4 - \frac{40}{3} + 12\right)\right]$

$= -6\pi\left(-\frac{5}{12}\right) = \frac{5}{2}\pi$

4. Use cylindrical shells to find the volume generated by rotating the region by $y = x^3$, y=8, x=0; about x=7 to set up the integral and find the volume

Shell has radius of $7 - x$ + circumference $2\pi(7-x)$

$V = \int_0^2 2\pi(7-x)(8-x^3)\,dx$

$= 2\pi \int_0^2 (x^4 - 7x^3 - 8x + 56)\,dx$

$= 2\pi \left[\frac{1}{5}x^5 - \frac{7}{4}x^4 - 4x^2 + 56x\right]_0^2$

$= 2\pi\left(\frac{32}{5} - 28 - 16 + 112\right) = 2\pi\left(\frac{372}{5}\right) = \frac{744\pi}{5}$