Sequences Worksheet

1. $a_n = n^2 + 5$
   a. $a_1 = (1)^2+5 = 6$
   b. $a_2 = (2)^2+5 = 9$
   c. $a_3 = (3)^2+5 = 14$
   d. $a_4 = (4)^2+5 = 21$
   e. $a_5 = (5)^2+5 = 30$

2. $a_n = \frac{1}{n}$, $n = 1$ to $n = \infty$

$$\left[1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots\right]$$

$a_1 = \frac{1}{1} = 1$

$a_2 = \frac{1}{2}$

$a_3 = \frac{1}{3}$

$a_4 = \frac{1}{4}$

$a_5 = \frac{1}{5}$

3. $a_n = \frac{(-1)^n}{2^n}$
   a. $a_0 = \frac{(-1)^0}{2^0} = \frac{1}{1} = 1$
   b. $a_1 = \frac{(-1)^1}{2^1} = -\frac{1}{2}$
   c. $a_2 = \frac{(-1)^2}{2^2} = \frac{1}{4}$
   d. $a_3 = \frac{(-1)^3}{2^3} = -\frac{1}{8}$
   e. $a_4 = \frac{(-1)^4}{2^4} = \frac{1}{16}$

4. $a_n = \frac{n}{2^n}$, $n = 0$ to $n = \infty$

$$\left[0, \frac{1}{2}, \frac{1}{2}, \frac{3}{8}, \frac{1}{4}, \ldots\right]$$

$a_0 = \frac{0}{2^0} = 0$

$a_1 = \frac{1}{2^1} = \frac{1}{2}$

$a_2 = \frac{2}{2^2} = \frac{2}{4} = \frac{1}{2}$

$a_3 = \frac{3}{2^3} = \frac{3}{8}$

$a_4 = \frac{4}{2^4} = \frac{4}{16} = \frac{1}{4}$

$a_0, a_1, a_2, a_3, a_4, a_5, \ldots$

5. $\left[1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \frac{1}{11}, \ldots\right]$

$b_1, b_2, b_3, b_4, b_5, b_6, \ldots$

$a_n = \frac{1}{2n+1} \rightarrow$

$\frac{1}{2(0)+1} = \frac{1}{1} = 1$

$\frac{1}{2(1)+1} = \frac{1}{3}$

$\frac{1}{2(2)+1} = \frac{1}{5}$

$\frac{1}{2(3)+1} = \frac{1}{7}$

$\frac{1}{2(4)+1} = \frac{1}{9}$

$\frac{1}{2(5)+1} = \frac{1}{11}$

$b_n = \frac{1}{2n-1} \rightarrow$

$\frac{1}{2(1)-1} = \frac{1}{1} = 1$

$\frac{1}{2(2)-1} = \frac{1}{3}$

$\frac{1}{2(3)-1} = \frac{1}{5}$

$\frac{1}{2(4)-1} = \frac{1}{7}$

$\frac{1}{2(5)-1} = \frac{1}{9}$

$\frac{1}{2(6)-1} = \frac{1}{11}$

6. $a_n = \frac{1}{2^n}$, $n = 1$ to $n = \infty$; $b_n = \frac{n}{n+4}$, $n = 5$ to $n = \infty$

a: $\lim_{n\to\infty} \frac{1}{2^n} \rightarrow$ goes to 0

b: $\lim_{n\to\infty} \frac{n}{n+4} \rightarrow$ goes to 1

$a_n + b_n \rightarrow 0 + 1 = 1$

7. $a_0 = -2,\ a_{n+1} = 5a_n + \frac{1}{4}$

a. $a_1 = 5a_0 + \frac{1}{4} \Rightarrow 5(-2) + \frac{1}{4} = -\frac{39}{4}$

b. $a_2 = 5a_1 + \frac{1}{4} \rightarrow 5\left(-\frac{39}{4}\right) + \frac{1}{4} = -\frac{97}{2}$

c. $a_3 = 5a_2 + \frac{1}{4} \rightarrow 5\left(-\frac{97}{2}\right) + \frac{1}{4} = -\frac{969}{4}$

d. $a_4 = 5a_3 + \frac{1}{4} \rightarrow 5\left(-\frac{969}{4}\right) + \frac{1}{4} = -1211$

e. $a_5 = 5a_4 + \frac{1}{4} \rightarrow 5(-1211) + \frac{1}{4} = -\frac{24219}{4}$

8. $a_0 = -1,\ a_{n+1} = \left(-\frac{1}{2}\right) a_n$

$a_{0+1} = a_1 = -\frac{1}{2} a_0 = -\frac{1}{2}(-1) = \frac{1}{2}$

$a_{1+1} = a_2 = -\frac{1}{2} a_1 = -\frac{1}{2}\left(\frac{1}{2}\right) = -\frac{1}{4}$

$a_{2+1} = a_3 = -\frac{1}{2} a_2 = -\frac{1}{2}\left(-\frac{1}{4}\right) = \frac{1}{8}$

$a_{3+1} = a_4 = -\frac{1}{2} a_3 = -\frac{1}{2}\left(\frac{1}{8}\right) = -\frac{1}{16}$

$a_{4+1} = a_5 = -\frac{1}{2} a_4 = -\frac{1}{2}\left(-\frac{1}{16}\right) = \frac{1}{32}$

9. $a_n = -\frac{(-8)^n}{3}$

a. $a_1 = -\frac{(-8)^1}{3} = \frac{8}{3}$

b. $a_2 = -\frac{(-8)^2}{3} = -\frac{64}{3}$

c. $a_3 = -\frac{(-8)^3}{3} = \frac{512}{3}$

d. $a_4 = -\frac{(-8)^4}{3} = -\frac{4096}{3}$

e. $a_5 = -\frac{(-8)^5}{3} = \frac{32768}{3}$

10. $a_n = \log\left(\frac{n^2}{n^2+1}\right), n = 0$ to $n = \infty$

$[-0.301, -0.097, -0.046, -0.026, -0.017, \dots]$

$a_1 = \log\left(\frac{1^2}{1^2+1}\right) = \log\left(\frac{1}{2}\right) = -0.301$

$a_2 = \log\left(\frac{2^2}{2^2+1}\right) = \log\left(\frac{4}{5}\right) = -0.097$

$a_3 = \log\left(\frac{3^2}{3^2+1}\right) = \log\left(\frac{9}{10}\right) = -0.046$

$a_4 = \log\left(\frac{4^2}{4^2+1}\right) = \log\left(\frac{16}{17}\right) = -0.026$

$a_5 = \log\left(\frac{5^2}{5^2+1}\right) = \log\left(\frac{25}{26}\right) = -0.017$