# Exam 2 Prep- Worksheet 1

- Integration by parts
  - Indefinite integrals
    - $\int \frac{\ln(x)}{x^3} dx$

$u = \ln(x)$ $v = -\frac{1}{2x^2}$

$du = \frac{1}{x}dx$ $dv = \frac{1}{x^3}dx$

$$= \ln(x)\left(-\frac{1}{2x^2}\right) - \int -\frac{1}{2x^2}\left(\frac{1}{x}\right)dx = -\frac{\ln(x)}{2x^2} + \frac{1}{2}\int \frac{1}{x^3}dx = -\frac{\ln(x)}{2x^2} + \frac{1}{2}\left(-\frac{1}{2x^2}\right) + c$$

$$= -\frac{\ln(x)}{2x^2} - \frac{1}{4x^2} + C$$

  - Definite integrals
    - $\int_0^3 x\sqrt{x+1}\, dx$

$u = x$ $v = \frac{2(x+1)^{3/2}}{3}$

$du = dx$ $dv = \sqrt{x+1}\,dx$

$$= x\left[\frac{2(x+1)^{3/2}}{3}\right]_0^3 - \int_0^3 \frac{2}{3}(x+1)^{3/2}dx = \frac{2}{3}x(x+1)^{3/2}\Big|_0^3 - \frac{2}{3}\cdot\frac{2}{5}(x+1)^{5/2}\Big|_0^3$$

$$= \frac{2}{3}x(x+1)^{3/2}\Big|_0^3 - \frac{4}{15}(x+1)^{5/2}\Big|_0^3 = \frac{2}{3}\left[3(3+1)^{3/2} - 0\right] - \frac{4}{15}\left[(3+1)^{5/2} - 1\right]$$

$$= \frac{2}{3}(24) - \frac{4}{15}(31) = \frac{116}{15}$$

  - Two parts
    - $\int_0^1 x^2 e^{5x} dx$

$u = x^2$ $v = \frac{1}{5}e^{5x}$

$du = 2x\,dx$ $dv = e^{5x}dx$

$$= x^2\left(\frac{1}{5}e^{5x}\right)\Big|_0^1 - \int_0^1 2x\left(\frac{1}{5}e^{5x}\right)dx = \frac{x^2e^{5x}}{5}\Big|_0^1 - \frac{2}{5}\int_0^1 xe^{5x}dx$$

$u' = x$ $v' = \frac{1}{5}e^{5x}$

$du' = dx$ $dv' = e^{5x}dx$

$$= \frac{x^2e^{5x}}{5}\Big|_0^1 - \frac{2}{5}\left[x\left(\frac{1}{5}e^{5x}\right)\Big|_0^1 - \int_0^1 \frac{1}{5}e^{5x}dx\right] = \frac{x^2e^{5x}}{5}\Big|_0^1 - \frac{2}{5}\left[\frac{xe^{5x}}{5}\Big|_0^1 - \frac{1}{5}\left(\frac{1}{5}e^{5x}\right)\Big|_0^1\right]$$

$$= \left\{\frac{1^2e^5}{5} - 0\right\} - \frac{2}{5}\left\{\left[\frac{e^5}{5} - 0\right] - \frac{1}{25}\left[e^5 - 1\right]\right\} = \frac{e^5}{5} - \frac{2e^5}{25} + \frac{2}{125}(e^5 - 1)$$

$$= \frac{25e^5 - 10e^5 + 2e^5 - 2}{125} = \frac{17e^5 - 2}{125}$$

- Trig integrals
  - $\int sin^k(x)cos^j(x)\,dx$
    - $\int sin^2(x)cos^2(x)dx$

$$= \int\left(\frac{1-\cos(2x)}{2}\right)\left(\frac{1+\cos(2x)}{2}\right)dx = \frac{1}{4}\int\left(1-\cos^2(2x)\right)dx = \frac{1}{4}\left(x - \int\cos^2(2x)dx\right)$$

$$= \frac{1}{4}\left(x - \int\left(\frac{1+\cos(4x)}{2}\right)dx\right) = \frac{1}{4}\left[x - \frac{1}{2}\int(1+\cos(4x))dx\right] = \frac{1}{4}\left[x - \frac{1}{2}\left(x + \frac{1}{4}\sin(4x)\right)\right] + C$$

$$= \frac{1}{4}\left[\frac{1}{2}x - \frac{1}{8}\sin(4x)\right] + C = \frac{1}{8}x - \frac{1}{32}\sin(4x) + C$$

  - $\int tan^k(x)sec^j(x)\,dx$
    - $\int tan^3(x)sec^6(x)dx$

$$\int \tan^3(x)\sec^4(x)\sec^2(x)dx = \int\tan^3(x)(\sec^2(x))^2\sec^2(x)dx = \int\tan^3(x)\left[\tan^2(x)+1\right]^2\sec^2(x)dx$$

$u = \tan(x)$
$du = \sec^2(x)dx$

$$= \int u^3[u^2+1]^2du = \int u^3(u^4+2u^2+1)du = \int u^7+2u^5+u^3\,du = \frac{u^8}{8} + \frac{2u^6}{6} + \frac{u^4}{4} + C$$

$$= \frac{1}{8}\tan^8(x) + \frac{1}{3}\tan^6(x) + \frac{1}{4}\tan^4(x) + C$$