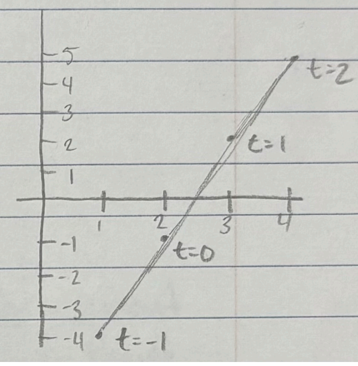
Curves Defined by Parametric Equations

- x & y are continuous functions of t on an interval I, x=x(t) & y=y(t)

1. find the x & y values then sketch a graph
   a. $x(t) = t + 2$ & $y(t) = 3t - 1$, [-1,2]

A) $x(t) = t+2$ & $y(t) = 3t-1$ $[-1,2]$

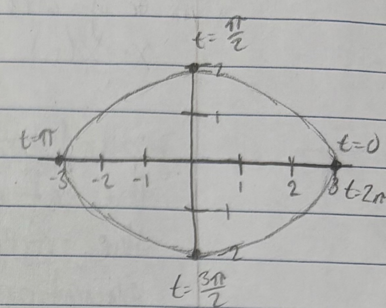
t	$x(t)=t+2$	$y(t)=3t-1$	(x,y)
-1	$x(-1)=(-1)+2=1$	$y(-1)=3(-1)-1=-4$	$(1,-4)$
0	$x(0)=0+2=2$	$y(0)=3(0)-1=-1$	$(2,-1)$
1	$x(1)=1+2=3$	$y(1)=3(1)-1=2$	$(3,2)$
2	$x(2)=2+2=4$	$y(2)=3(2)-1=5$	$(4,5)$



   b. $x(t) = 3\cos t$ & $y(t) = 2\sin t$, $[0,2\pi]$

c) $x(t) = 3\cos(t)$ & $y(t) = 2\sin(t)$ $[0, 2\pi]$

t	$x(t)=3\cos(t)$	$y(t)=2\sin(t)$	(x,y)
0	$x(0)=3\cos(0)=3$	$y(0)=2\sin(0)=0$	$(3,0)$
$\pi/2$	$x(\frac{\pi}{2})=3\cos(\frac{\pi}{2})=0$	$y(\frac{\pi}{2})=2\sin(\frac{\pi}{2})=2$	$(0,2)$
π	$x(\pi)=3\cos(\pi)=-3$	$y(\pi)=2\sin(\pi)=0$	$(-3,0)$
$3\pi/2$	$x(\frac{3\pi}{2})=3\cos(\frac{3\pi}{2})=0$	$y(\frac{3\pi}{2})=2\sin(\frac{3\pi}{2})=-2$	$(0,-2)$
2π	$x(2\pi)=3\cos(2\pi)=3$	$y(2\pi)=2\sin(2\pi)=0$	$(3,0)$



2. Find the equation of the tangent line at $t = \frac{\pi}{6}$ then find the x & y values and sketch a graph

   a. $x(t) = 4\cos(t)$ & $y(t) = 3\sin(t)$, $[0, 2\pi]$

A) $x(t) = 4\cos(t)$ $\quad y(t) = 3\sin(t)$ $\quad [0, 2\pi]$

$x'(t) = -4\sin(t)$ $\quad y'(t) = 3\cos(t)$

$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{3\cos(t)}{-4\sin(t)} = -\frac{3}{4}\cot(t)$

B) $m = \left.\frac{dy}{dt}\right|_{t=\frac{\pi}{6}} = -\frac{3}{4}\cot\left(\frac{\pi}{6}\right) = -\frac{3}{4}\sqrt{3} = -\frac{3\sqrt{3}}{4}$

$\left.\begin{array}{l} x\left(\frac{\pi}{6}\right) = 4\cos\left(\frac{\pi}{6}\right) = 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3} \\ y\left(\frac{\pi}{6}\right) = 3\sin\left(\frac{\pi}{6}\right) = 3\left(\frac{1}{2}\right) = \frac{3}{2} \end{array}\right\} \left(2\sqrt{3}, \frac{3}{2}\right)$

$y - y_1 = m(x - x_1) \Rightarrow$ point slope

$y - \frac{3}{2} = -\frac{3\sqrt{3}}{4}(x - 2\sqrt{3}) = -\frac{3\sqrt{3}}{4}x + \frac{3\sqrt{3}}{\cancel{4}_2} \cdot \cancel{2}\sqrt{3}$

$y - \frac{3}{2} = -\frac{3\sqrt{3}}{4}x + \frac{9}{2} \Rightarrow y = -\frac{3\sqrt{3}}{4}x + 6$

C) $x(t) = 4\cos(t)$ $\quad y(t) = 3\sin(t)$, $[0, 2\pi]$

t	x	y
0	4	0
$\frac{\pi}{2}$	0	3
π	-4	0
$\frac{3\pi}{2}$	0	-3
2π	4	0