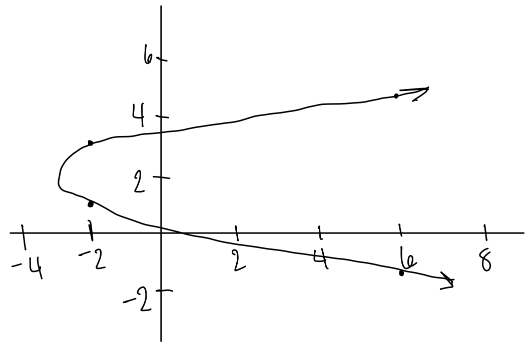
Test Prep 5 Worksheet 2

1. $x = t^2 - 3, y = t + 2, -3 \leq t \leq 3$, sketch the curve using parametric equations and eliminate the parameter to find the Cartesian equation of the curve for $-1 \leq y \leq 5$

t	-3	-1	1	3
x	6	-2	-2	6
y	-1	1	3	5

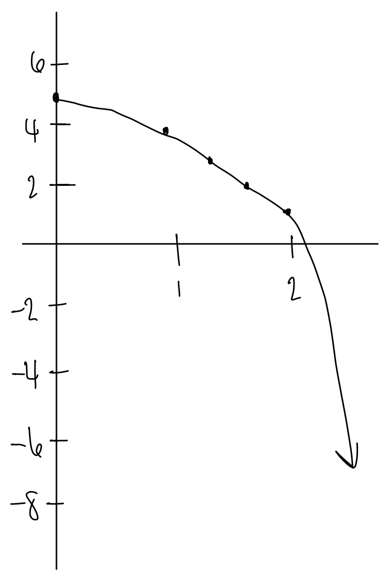


$y = t + 2 \rightarrow t = y - 2$

$x = t^2 - 3 \rightarrow (y-2)^2 - 3 = y^2 - 4y + 4 - 3 \rightarrow x = y^2 - 4y + 1, \ -1 \leq y\ 5$

2. $x = \sqrt{t}, y = 5 - t$, sketch the curve of the parametric equations as t increases and eliminate the parameter to find the cartesian equation of the curve for $x \geq 0$

t	0	1	2	3	4
x	0	1	1.4	1.7	2
y	5	4	3	2	1



$x = \sqrt{t} \rightarrow t = x^2$

$y = 5 - t \rightarrow 5 - x^2 \rightarrow y = 5 - x^2, \ 0 \leq x \leq 2$

3. $x = 5t^3 + 3t, y = 4t - 6t^2$, find $\frac{dx}{dt}, \frac{dy}{dt}, \frac{dy}{dx}$

$\frac{dx}{dt} = 15t^2 + 3$ $\frac{dy}{dt} = 4 - 12t$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4-12t}{15t^2+3}$

4. $x = t^3 + 1, y = t^{10} + t; t = -1$, find the equation of the tangent to the curve

$\frac{dx}{dt} = 3t^2$ $\frac{dy}{dt} = 10t^9 + 1$ $\frac{dy}{dt} = \frac{10t^9+1}{3t^2}$

when $t = -1$ $x = 0$ + $y = 0$ $\frac{dy}{dx} = -\frac{9}{3} = -3$

$y - y_1 = \frac{dy}{dx}(x - x_1) \Rightarrow y - 0 = -3(x - 0) \Rightarrow y = -3x$

5. $(-8,8)$, find the polar coordinates given cartesian coordinates

$r^2 = x^2+y^2 \rightarrow r = \sqrt{(-8)^2+(8)^2} \cong 11.31$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{8}{-8}\right) = \underset{(QII)}{-45+180} = 135°$

$(11.31, \angle 135°)$

6. $r = 3\cos\theta$, find the cartesian equation and identify the curve

$r^2 = x^2+y^2$ $\quad$ $r^2 = 3\overbrace{r\cos\theta}^{x} \rightarrow x^2+y^2 = 3x \Rightarrow x^2-3x+\frac{9}{4}+y^2 = \frac{9}{4} \rightarrow \left(x-\frac{3}{2}\right)^2+y^2 = \frac{9}{4}$

$r = \frac{3}{2}$ centered @ $\left(\frac{3}{2}, 0\right)$

$r^2 = 3r\cos\theta \rightarrow r(r-3\cos\theta)=0$

$r=0$ + $3\cos\theta \rightarrow$ circle

7. $r = \sqrt{18\theta}, 0 \le \theta \le \frac{\pi}{2}$, find the area of the region bonded by the given curve

$$A = \int_0^{\pi/2} \frac{1}{2} r^2 d\theta = \int_0^{\pi/2} \frac{1}{2}\left(\sqrt{18\theta}\right)^2 d\theta = \int_0^{\pi/2} \frac{1}{2}(18\theta)d\theta = \int_0^{\pi/2} 9\theta\, d\theta$$

$$= \left[\frac{9}{2}\theta^2\right]_0^{\pi/2} = \frac{9}{2}\left(\frac{\pi}{2}\right)^2 = \frac{9}{8}\pi^2$$

8. $r = 3\cos\theta, 0 \le \theta \le \pi$, find the exact length of the polar curve

$$L = \int_0^{\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta \qquad \frac{dr}{d\theta} = -3\sin\theta$$

$$L = \int_0^{\pi} \sqrt{(3\cos\theta)^2 + (-3\sin\theta)^2}\, d\theta = \int_0^{\pi} \sqrt{9\cos^2\theta + 9\sin^2\theta}\, d\theta = \int_0^{\pi} \sqrt{9(\cos^2\theta + \sin^2\theta)}\, d\theta$$

$$= \int_0^{\pi} \sqrt{9}\, d\theta = 3\theta\Big]_0^{\pi} = 3\pi$$