Calculus II Worksheet
Trig Substitutions ($\int\sqrt{a^2-x^2}$)

- Sub: $x=a\sin(\Theta)$
  - $Dx=a\cos(\Theta)d\Theta$
  - $\sin(\Theta)=x/a$
  - $\cos(\Theta)\geq 0$
  - $\Theta=\sin^{-1}(x/a)$

1. $\int\sqrt{(4-x^2)}dx$ $\quad a^2=4 \Rightarrow a=2$

$x=2\sin(\theta)$ $\quad \sin(\theta)=\frac{x}{2}$

$dx=2\cos(\theta)d\theta$ $\quad \theta=\sin^{-1}\left(\frac{x}{2}\right)$

$$\int\sqrt{4-(2\sin(\theta))^2}\cdot 2\cos(\theta)\,d\theta$$

$$\int\sqrt{4-4\sin^2(\theta)}\cdot 2\cos(\theta)\,d\theta$$

$$\int\sqrt{4(1-\sin^2(\theta))}\cdot 2\cos(\theta)\,d\theta$$

$$\int 2\sqrt{\cos^2(\theta)}\cdot 2\cos(\theta)\,d\theta$$

$$\int 4|\cos(\theta)|\cos(\theta)\,d\theta$$

$$\int 4\cos^2(\theta)\,d\theta$$

$$\int 4\left[\frac{1+\cos(2\theta)}{2}\right]d\theta$$

$$2\int 1+\cos(2\theta)\,d\theta$$

$$2\left[\theta+\frac{1}{2}\sin(2\theta)\right]+C$$

$$2\left[\sin^{-1}\left(\frac{x}{2}\right)+\frac{1}{2}(2\sin(\theta)\cos(\theta))\right]+C$$

$$2\left[\sin^{-1}\left(\frac{x}{2}\right)+\left(\frac{x}{2}\cdot\frac{\sqrt{4-x^2}}{2}\right)\right]+C$$

2. $\int\sqrt{(576-36x^2)} = 6\int\sqrt{16-x^2}\,dx$

$a=4 \quad x=4\sec\theta \rightarrow \sec\theta=\frac{x}{4}$

$dx=4\sec\theta\tan\theta\,d\theta \quad \theta=\sec^{-1}\frac{x}{4}$

$6\int\sqrt{16-(4\sec(\theta))^2}\cdot 4\sec(\theta)\tan(\theta)\,d\theta$

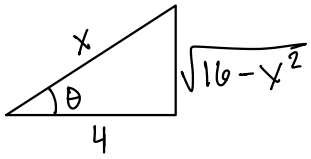
$=24\int\sqrt{16(1-\sec^2(\theta))}\cdot\sec(\theta)\tan(\theta)\,d\theta$

$=96\int\sec(\theta)\tan^2(\theta)\,d\theta$

$=96\int\sec^3(\theta)\sec(\theta)\,d\theta$

$=96\left[\frac{1}{2}\sec(\theta)\tan(\theta)-\frac{1}{2}\ln\left|\sec(\theta)+\tan(\theta)\right|\right]+C$

$=48\left[\frac{x}{4}\cdot\frac{\sqrt{16-x^2}}{4}-\ln\left|\frac{x}{4}+\frac{\sqrt{16-x^4}}{4}\right|\right]+C$



3. $\int\frac{x^2}{\sqrt{(64(36-x^2))}}dx$ close to $\sqrt{a^2-x^2}$

$\int\frac{x^2}{8\sqrt{36-x^2}}dx \quad a=6 \quad x=6\sin(\theta) \quad dx=6\cos(\theta)\,d\theta$

$\sin(\theta)=\frac{x}{6} \quad \theta=\sin^{-1}\left(\frac{x}{6}\right)$

$\frac{1}{8}\int\frac{(6\sin(\theta))^2(6\cos(\theta))}{\sqrt{36-(6\sin(\theta))^2}}d\theta$

$=\frac{1}{8}\int\frac{36\sin^2(\theta)(6\cos(\theta))}{6\sqrt{\cos^2(\theta)}}d\theta$

$=\frac{36}{8}\int\frac{\sin^2(\theta)\cos(\theta)}{\cos(\theta)}d\theta$

$=\frac{36}{8}\int\sin^2(\theta)\,d\theta$

$=\frac{36}{8}\left[\frac{1}{2}\theta-\frac{1}{4}\sin(2\theta)\right]+C$

$=\frac{36}{8}\left[\frac{1}{2}(\sin^{-1}(\frac{x}{6}))-\frac{1}{4}\sin(2(\sin^{-1}(\frac{x}{6})))\right]+C=\frac{9}{4}\sin^{-1}\left(\frac{x}{6}\right)-\frac{9}{8}\sin(2\sin^{-1}\left(\frac{x}{6}\right))+C$